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BLACK-SCHOLES MODEL AND STOCK PRICE VOLATILITY ESTIMATION IN THE NIGERIAN CAPITAL MARKET

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ABSTRACT

The Black-Scholes option valuation model with input variables: stock price and stock price volatility, present value of option strike price and time to expiration of option make possible the estimation of stock prices and stock price volatility of underlying assets of options when option values are available. A comparative analysis of stock price volatility estimations from the Black-Scholes model with that based on market indices using the one-tailed F-test concerning two population variances with primary data from four quoted firms with options attached to their underlying assets (shares) show that stock price volatilities estimated using the Black-Scholes model are far lower than volatilities estimated for the sampled firms using stock market indices, implying the inability of the model to estimate actual stock price volatility in the Nigerian capital market as it has done in Europe. The extreme volatility values from using market indices of the sampled firms' stocks, a reflection of persistent stock price increases, imply the existence of stock price value movements beyond stock fundamentals (which are more comprehensive in the Black-Scholes model than that resulting in actual market volatility) suggesting the existence of stock price manipulations based on manipulatable input variables of existing models compared to the Black-Scholes model with varied less-manipulatable input variables, necessitating improved regulation by capital market regulators to curtail manipulated stock price volatility in the Nigerian capital market. If these volatilities subsist, investors should request for higher expected rate of returns on their stock investments as required by the Capital Asset Pricing Model (CAPM) when β (a measure of variability of returns including capital gains which is affected by the volatility of stock prices) is high to entrench and sustain security valuations in the Nigerian capital market based on all information about the security and market fundamentals.

***Key Words**:* Black-Scholes model, capital market, stock price estimation, stock price volatility

INTRODUCTION

Share price volatility, an input variable in the Black-Scholes (1973) option pricing model, is seen to be systematically related to the option's exercise price and time to expiration. This assertion was confirmed by Dumas et al (1995) from their study of a sample of Standards and Poors (S&P) 500 index options during the period 1988-1993. This systematic behaviour of share price volatility to Derman and kani (1994), Dupire (1994) and Rubinstein (1994) is driven by the fact that volatility rate of option's underlying asset's return varies with the level of assets price and time. The superiority of the Black-Scholes (1973) approach to stock price volatility determination over others was stressed by Chiras and Manaster (1978) and Fleming et al (1995), as the efficient market hypothesis in its semi-strong form according to Fama (1970) reflects all information about the security (options inclusive where in existence) which Black-Scholes (1973) model provides. To Schonbucher (1998), these volatilities are implied and may be static or changing and are observable from the market.

In application, Van Zanten (2009) noted that ensuring that all information are included in either stock price determination or its volatility determination, the Black-Scholes (1973) seems preferable as it is simple, mathematically tractable, considers no arbitrage with explicit expression for basic options and hedging strategies which can use both PD (partial differential) techniques and stochastic analysis tools. Reconciling this model and the risk-neutral expected value model, Ekstrom and Tysk (2011) observed that there exists a connection between these approaches to option value determination provided by the Feynman-Kac theorem which states that a classical solution to a linear parabolic PDE (partial differential equation) has a stochastic representation in terms of an expected value. In the standard Black-Scholes (1973) model, Ekstrom and Tysk (2011) added that a standard logarithmic change of variables transforms the Black-Scholes (1973) equation into an equation with constant coefficients i.e.

$C_o = S_o N(d_1) - Pv\,(K)\,N(d_1 - \sigma\sqrt{T})$ …………………..1

where $d_1 = \{In\{So/Pv(K)\}/\sigma\sqrt{T}\}$……………..….2

developed for the valuation of options (C_o) where input variables: S_o $N(d_1)$, the cost of the shares needed in tracking portfolio and Pv (K) $N(d_1 - \sigma\sqrt{T})$, the number of naira borrowed at risk-free rate for the purchase of the option to be valued; and σ the volatility of share prices, if rearranged with a constant σ gives

$\sigma = \{(C_o - S_o Nd1/Pv(k)N)\text{-}d1\}/\sqrt{T}$.……………..3

where So= value of the stock stripped of dividend to expiration

Co=price of the option

N(d1)= probability the number of shares lies between zero and one

Pv(K)= present value strike price of the option

T= expiration of the option

σ=standard deviation of underlying asset (shares)

Using equation 3 and given option value (C_o), stock price volatility can be determined for stocks with attached options.

To Grinblatt and Titman (2003), equation 1 holds if the variance and the risk-free rate when they change does so in predictable manner; if otherwise, they noted no risk-free hedge exists between the stock values, its volatility and the option value. The model assumes that the logarithm of the return on the underlying stock is normally distributed. Thus at any given finite period, the stock price may be close to zero, though with low probability. Furthering, they observed that subtracting a finite dividend from the low value would result in a negative ex-dividend stock price. Analysts of the Black-Scholes (1973) model (Ekstrom and Tysk, 2011; Grinblatt and Titman, 2003; Fouque et al, 2000; Sircar et al, 2000; and Schonbucher, 1998) concluded that as the volatility of a stock price increases, the value of the call and put option on this underlying assets (C_o) increases. Thus as increased volatility spreads, the distribution of the future stock price 'fattens' up both tails of the distribution. This being the case, an estimation of σ of a stock will give investors the likely market value of stocks bearing options when the Black-Scholes model is re-arranged in favour of stock prices with attached options, an alternative to the Gordon's (1959) dividend growth model for stock price determination. Though stock price volatility can be estimated using market values which seems deceptive in the Nigerian capital market due to the existence of much stock price manipulations which are made excusable, the Black-Scholes (1973) model provides a reliable and less manipulative means of determining stock price volatility (from its fairly complex nature) which itself is an input variable to stock price determination.

Objective of the study

The purpose of this study is to determine the predictive strength of the Black-Scholes (1973) model in stock price volatility estimation in the Nigerian capital market.

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

Theoretical framework

The Black-Scholes model developed for valuation of European options gives the price of option in terms of other known quantities: exercise price of the option, time to maturity and current stock price and its volatility. This model is hinged on the assumption that the time interval between observations is very small, and that log prices follow a random walk with normally distributed innovations which are not affected by any linear drift in the random walk. An important input into the Black-Scholes model, σ, the standard deviation of the stock's continuously compounded rate of return (Δlog price divided by the length of time lapse) is the standard deviation of the innovations in the random walk if the time interval between observations is sufficiently small, which to Haugh (2010) make σ the volatility of the stock price.

LITERATURE REVIEW

The Black-Scholes model

The Black-Scholes (1973) option valuation model: $C_o = S_o N(d_1) - Pv(K) N(d_1 - \sigma\sqrt{T})$ has input variables: the current market price of the underlying asset of the option, S_o (shares), standard deviation of the share values (σ), expiration date of the option valued (T), present value of the strike price of the option (Pv(K) and cumulative standard normal distribution function $N(d_1)$. Rearranging this model with σ as the subject of the equation given the option value (C_o), makes possible the estimation of the volatility of prices of stocks underlying options. Findings by Macbeth and Merville (1979) using daily closing prices to study actively traded options on six stocks showed their volatilities to be inversely related to the strike price of the option (using the original Black-Scholes model). Using transaction data, Rubinstein (1985) found that volatility of stock prices of underlying asset's of options increases when the expiration date of the attached option is shorter. Additional findings from analysed transaction data from August 1976 to October 1997 by Rubinstein (1985) showed that stock price volatility increases when attached option values are lower.

Extensions to the original Black-Scholes (1973) model by Haugh (2010) made the model useable for pricing commodity and foreign exchange derivatives. Resultant models: local and stochastic volatility models, jump-diffusions and others built from Levy process according to Haugh (2010) can be used to price exotic securities. The parameters for estimating option values (C_o) in the original Black-Scholes (1973) model according to Gross (2006), are observable except the volatility which needs to be estimated using the historical volatility or implied volatility; but where C_o is given, the rearranged model can estimate volatility (σ) of the underlying asset of the option.

The Black-Scholes (1973) formula was derived under the assumption that the time interval between observations is very small, and that the log prices follow a random walk with normally distributed innovations. Allegations of the unrealistic nature of the assumptions underlying this model are rife as the geometric Brownian motion model which implies that the series of first differences of the log prices must be correlated.

Haugh (2010) argued that in reality, there exists a small but statistically significant correlation in the differences of the logs at short time lags; noting that results from observations of Standards and Poors (S&P) 500 daily movements in option prices from July 1962 to December 1995 substantiates this. The normality of distributed returns assumption is seen as unrealistic as returns are leptokurt. The constancy of σ assumption was faulted by findings from analysed financial data by Haugh (2010) as results show changes in σ overtime. But any violations of these underlying assumptions to Hull (2000) make the model invalid.

Black-Scholes equation, stock price determination and stock price volatility

The Black-Scholes (1973) European option valuation model has over the years made possible the valuation of the underlying asset of the option (S_o) and the volatility (σ) of the stock values given the option underlying asset value as S_o and σ which are input variables into the model. Grinblatt and Titman (2003) posits that the values of both the call and put options valued using the model increase as the volatility of the stock price increases; noting that this creates conflict between equity holders and debt holders with attendant financial implications for corporate financial objectives and functions of portfolio managers. Intuitively, this finding means a fattening up of tails of the distribution of future stock prices. Extending the Black-Scholes (1973) model, Dumas et al (1995) made possible the determination of the current price of underlying price of stock of an option as well as the future price using the Black-Scholes (1973) model (which had hitherto been derived using the Gordon, 1959, growth model). This is made possible with the transformation of observed spot prices for cash index and option series into future price. Dupire (1994), Rubinstein (1994) and Derman and Kani (1994) noted that systematic behavior between variables in the Black-Scholes (1973) model is caused by the fact the volatility rate of asset return (which measures the volatility of stock prices) varies with the level of asset price and time.

Volatility of stock incomes (capital gains and dividends) are input variables in determining β in the Capital Asset Pricing Model (CAPM). Thus the higher the volatility as directly determined using the Black-Scholes (1973) model, the higher will be the expected return from stock investment. The expected return, which is an input variable in the Gordon's (1959) growth model of share price determination, influences the value of the stock; which seems an indirect approach to stock price determination using the Black-Scholes (1973) model. Given the option value (C_o) and the volatility of the price of the underlying asset of the option, the model can directly value the underlying asset (both current and future prices) provided there is a transformation of observed spot prices for cash index and option series into forward prices as required by Dumas et al (1995), giving a novel approach to stock price determination aside the conventional Gordon's (1959) growth model, the price/earnings ratio and book value of assets valuation model.

METHODOLOGY

Sample selection

Samples for this study are four most traded non-bank shares (Nestle Foods Nig Plc, Guinness Nig Plc, Unilever Nig Plc and Cadbury Nig plc) with high incidences of price movements in both directions having options based on underlying shares of these firms. Bank shares were excluded because of the existence of externalities affecting their values and stagnancy of their values since the crash in the capital market.

Model justification

The Black-Scholes model:

$C_o = S_o N(d_1) - Pv(K) N(d_1 - \sigma\sqrt{T}$ …………...........1

where $d_1 = \{In\{So/Pv(K)\}/\sigma\sqrt{T}\}$ ………….........2

developed for the valuation of European options (C_o) requires input variables: S_o $N(d_1)$, the cost of the shares needed in tracking portfolio and Pv (K) $N(d_1 - \sigma\sqrt{T})$, the number of naira borrowed at risk-free rate for the purchase of the option to be valued. σ in equation 1 is the volatility of share prices. Determination of this volatility using the Black-Scholes model requires the re-arrangement of the equation, i.e.

$\sigma = \{(C_o - S_o Nd1/Pv(k)N) - d1\}/\sqrt{T}$.……………....3

where So= value of the stock stripped of dividend to expiration

Co= value of the option

N(d1)= probability the number of shares lies between zero and one

Pv(K)= present value of the strike price of the option

T= expiration of the option

Equation 3 given option value (C_o) will determine the volatility of the shares.

Since the aim of this study is to determine whether the volatility of share price (σ) computed using the Black-Scholes model is statistically lower or higher than/the same with share price volatility using the market indices, the one tailed F-test concerning two population variances is the ideal analytical model to determine this, where

$F = \{\{(n_1-1)S_1^2/\sigma_1^2\}/n_1-1\}/\{\{(n_2-1)S_2^2/\sigma_2^2\}/(n_2-1)\}$ with (n_1-1) and (n_2-1) degrees of freedom.

where n1= size of sample 1; S_1^2 =variance of sample 1; σ_1^2=variance of population 1; n_2=size of sample 2; S_2^2= variance of sample 2; and σ_2^2 = variance of population 2.

Data presentation

The Black-Scholes option valuation model for determining the volatility (σ) of stock values:

$\sigma = \{(C_o - S_o Nd_1/Pv(K)N) - d_1\}/\sqrt{T}$……………....1

require input variables: C_o, S_o, Nd_1, Pv(K), N, d_1 and $\sqrt{T}$ of the shares of the sampled firms: Nestle Foods Nig Plc, Guinness Nig Plc, Unilever Nig Plc and Cadbury Nig plc. σ values for these firms for 2010-2012 using market indices are shown in table 1.

Table 1: σ and σ^2 of stock prices of sampled firms using the Back-Scholes model and stock market indices.

Sampled firms' market prices within study period

Study period	Nestle Foods Nig Plc(N)	Cadbury Nig Plc(N)	Guinness Nig Plc(N)	Unilever Nig Plc(N)		Grand σ^2 (S^2)
December 2012	700.00	29.00	276.00	47.30		
January 2012	400.00	18.3	201.00	27.01		
December 2011	400.00	8.33	201.11	27.00		
January 2011	210.00	8.33	118.00	26.05		
December 2010	210.05	8.00	110.00	26.00		
January 2010	176	8.00	100.00	18.50		
σ based on market indices	198.44	8.65	8.65	9.70	94.72	8972.12
σ using the Black-Scholes model	0.1165	0.3226	0.1708	0.7890	0.93	0.30556

Source: computed from stock market indices and input variables for option valuation

σ for stock values of sampled firms were computed for the four firms using the Black-Scholes (1973) model and option values based on the underlying shares (C_o) and other required variables in equation 1. Actual σ values for the shares for comparison with σ from computation using the Black-Scholes model were σ computed using the actual price data of the four firm shares for 2010-2012 as shown in table 1.

Data analysis

A comparative analysis of volatility obtained from the Black-Scholes model with the market volatility using the F-test concerning two population variables will be done to ascertain whether volatility from the Black-Scholes model

(σ_1) is statistically lower or higher than/the same as actual volatility (σ_2) in share values. As the hypothesis is of the greater than or equal to type, we use the one tail test.

Let σ_1^2 (grand variance using the Black-Scholes model) and σ_2^2 (grand variance using actual stock market indices) be the variances of two populations which follow normal distribution. Thus

$H_o: \sigma_1^2 < \sigma_2^2$; and

$H_1: \sigma_1^2 \geq \sigma_2^2$

If the sample sizes are n_1 and n_2, the F-distribution is

$F = \{\{(n_1-1)S_1^2/\sigma_1^2\}/n_1-1\}/\{\{(n_2-1)S_2^2/\sigma_2^2\}/(n_2-1)\}$ with (n_1-1) and (n_2-1) degrees of freedom.

Since the objective of any test is to minimize type 1 error which is usually higher at the transition point between H_o and H_1 which Panneerslvan (2004) noted occurs when $\sigma_1^2 = \sigma_2^2$, he argued that the F distribution formula is reduced to the form:

$F = S_1^2/S_2^2$ with (n_1-1) and (n_2-1) degrees of freedom

if the condition holds; where the numerator of the degrees of freedom (n_1-1) and the denominator degrees of freedom (n_2-1) are the only parameters affecting its shape and size.

Therefore using data S_1^2 and S_2^2 from table 1, F=0.30556/8972.12 = 0.0000341.

The table F value with (2,2) degrees of freedom at the given significance level of 0.05 is 19.00. Since the computed F ratio (0.0000341) is less than the table F value (19.00), it falls in the acceptance region. Hence H_o is accepted. Therefore the volatility of stock prices using the Black-Scholes model is lower than the volatility of share market prices using market indices i.e. volatility of share prices determined using the Black-Scholes model is different from volatility using stock market indices.

DISCUSSION OF FINDINGS, CONCLUSIONS, POLICY IMPLICATIONS OF FINDINGS AND RECOMMENDATIONS

The result above show that computed volatility (σ) from using actual market indices of sampled stocks differ significantly from the volatility of stock prices obtained using the Black-Scholes (1973) model implying the inability of the model to predict stock price volatility in the Nigerian capital market which it has successfully done in European countries (Grinblatt and Titman, 2003). This may be attributed to the existence of fair level of the implicit perfect capital market assumption of the model which exists minimally in the Nigerian capital market. The inability of the Black-Scholes model to predict stock price volatility of shares of sampled firms substantiates findings by Osaze (2005) that much share price manipulations exist in the Nigerian capital market as share prices do not reflect fundamentals of share values which are required input variables for the Black-Scholes model.

The existence of high volatility in the actual share values reflects the high volatility in capital gains earnings with implicit high level of risk in capital gains earnings. To compensate investors for this risk, current shareholders' income (dividend) from share investments need to be high necessitating shareholders to demand high current naira dividend or payout ratios. The Capital Asset Pricing Model (CAPM) with high risk (β) value dependent on high actual σ of the security requires investors to demand high rate of on investment. Though the actual market values are on the upward trend, crash in market values of securities as experienced in 2008 and 2009 will result in huge capital losses; while current dividend desiring shareholders are rewarded with the improved current incomes. Had k_e in the Gordon's (1959) dividend growth model used in practice for stock price determination been derived from expected return (under the CAPM) obtained based on volatility determined using the Black-Scholes (1973) model which are seen to be lower, determined stock prices in the Nigerian capital market using the Gordon's (1959) model would have been lower (almost the post-crash values), eliminating the phantom pre-crash capital gains and eventual crash of stock values to values based on all information on the stock's fundamentals (including the attached options). Market regulators should eliminate price manipulations, mete severe punishments to culprits, and ensure that market values of shares reflect shares' fundamentals. Share values with traded derivatives' (options) values can be determined using the non-conventional Black-Scholes model as more variables fundamental to both the underlying asset (shares) and derivatives attached to it determine the market value of the share which seems less subject to manipulations as such will require manipulation both the share variables and derivatives' variables.

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